

Automatic Polytime Reductions of NP Problems into a Fragment of STRIPS

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Motivation

- for using planners, one needs to **come up** with **sound** PDDLs
- even if you know well the problem, it may not be easy to **translate** it into PDDL
- it would be nice to **automatically** translate problems described in high-level declarative language into PDDL

Our Contribution

- tool that **automatically** translates NP problems into PDDL
- problems **specified using logic** in declarative manner
- translation **runs in polytime**
- existence of plans for generated PDDLs can be **decided in NP**
- tool fully characterized by its **formal properties**

This Talk

- Descriptive Complexity Theory
- Tool
- Translations
- Experiments
- Discussion

Descriptive Complexity Theory

Branch of Complexity Theory that uses **logic instead of TMs** to characterize complexity classes

In Descriptive Complexity Theory (DCT):

- problem corresponds to collection of **finite structures**
- collection is the set of finite models for a **logic formula**
- complexity class (class of problems) corresponds to a **fragment of logic**

For example, NP equals all problems **definable** in the existential fragment of second-order logic ($SO\exists$)

Main results of DCT:

- P equals SO-Horn
- NP equals $SO\exists$ and coNP equals $SO\forall$
- Polynomial-time hierarchy (PH) equals SO
- PSPACE equals SO + Transitive Closure ($SO+TC$)

E.g., $PH = PSPACE$ iff TC does not add **expressivity** to SO

Example: SAT

$$\text{CNF } \varphi = \underbrace{(x_0 \vee \neg x_1 \vee x_2)}_{\text{clause 0}} \wedge \underbrace{(\neg x_0 \vee \neg x_2)}_{\text{clause 1}} \wedge \underbrace{(\neg x_0 \vee x_1)}_{\text{clause 2}}$$

Example: SAT

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Encoded with relations $P(x, y)$ and $N(x, y)$ interpreted as:

- $P(x, y)$ iff variable x appears **positive** in clause y
- $N(x, y)$ iff variable x appears **negative** in clause y

Example: SAT

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E.g., φ **encoded** with structure $\mathcal{A} = \langle |\mathcal{A}|, P^{\mathcal{A}}, N^{\mathcal{A}} \rangle$ where

- universe is $|\mathcal{A}| = \{0, 1, 2\}$
- interpretation of P is $P^{\mathcal{A}} = \{(0, 0), (2, 0), (1, 2)\}$
- interpretation of N is $N^{\mathcal{A}} = \{(1, 0), (0, 1), (2, 1), (0, 2)\}$

Example: SAT

$$\text{CNF } \varphi = \underbrace{(x_0 \vee \neg x_1 \vee x_2)}_{\text{clause 0}} \wedge \underbrace{(\neg x_0 \vee \neg x_2)}_{\text{clause 1}} \wedge \underbrace{(\neg x_0 \vee x_1)}_{\text{clause 2}}$$

Model: $\{x_0, x_1, \neg x_2\}$ encoded with **unary** relation $T(x)$ such that

- $T(x_0)$
- $T(x_1)$
- $\neg T(x_2)$

I.e., T has interpretation $\{0, 1\}$

Example: SAT

$$\text{CNF } \varphi = \underbrace{(x_0 \vee \neg x_1 \vee x_2)}_{\text{clause 0}} \wedge \underbrace{(\neg x_0 \vee \neg x_2)}_{\text{clause 1}} \wedge \underbrace{(\neg x_0 \vee x_1)}_{\text{clause 2}}$$

Extended structure $\langle |\mathcal{A}|, P^{\mathcal{A}}, N^{\mathcal{A}}, T \rangle$ is model of

$$(\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]$$

iff T encodes a model of φ

Example: SAT

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iff T encodes a model of φ

Hence, φ is **SATISFIABLE** iff \mathcal{A} is **model** of

$$\Phi = \underbrace{(\exists T)}_{\text{s.o.}\exists} (\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]$$

Example: SAT

$$\text{CNF } \varphi = \underbrace{(x_0 \vee \neg x_1 \vee x_2)}_{\text{clause 0}} \wedge \underbrace{(\neg x_0 \vee \neg x_2)}_{\text{clause 1}} \wedge \underbrace{(\neg x_0 \vee x_1)}_{\text{clause 2}}$$

Indeed, $\text{SAT} = \text{MOD}[\Phi]$

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Indeed, $\text{SAT} = \text{MOD}[\Phi]$

Meaning:

- for every satisfiable formula φ , its encoding $\mathcal{A}_\varphi \in \text{MOD}[\Phi]$
- for every $\mathcal{A} \in \text{MOD}[\Phi]$, \mathcal{A} encodes a satisfiable formula $\varphi_{\mathcal{A}}$

Example: 3-Colorability

Signature

- $E(x, y)$: undirected edge linking nodes x and y in the graph

Formula

Every node must be colored with single color; if two nodes are connected, their colors must be different

$$\begin{aligned} &(\exists R^1, G^1, B^1)(\forall x, y)[\\ &\quad (R(x) \vee G(x) \vee B(x)) \wedge \\ &\quad R(x) \rightarrow \neg(G(x) \vee B(x)) \wedge \\ &\quad G(x) \rightarrow \neg(R(x) \vee B(x)) \wedge \\ &\quad B(x) \rightarrow \neg(R(x) \vee G(x)) \wedge \\ &\quad E(x, y) \rightarrow \neg[(R(x) \wedge R(y)) \vee (G(x) \wedge G(y)) \vee (B(x) \wedge B(y))]] \end{aligned}$$

Example: Directed Hamiltonian Path

Signature

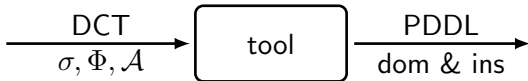
- $E(x, y)$: directed edge linking nodes x and y in the graph

Formula

A DHP is a sequence vertices such that there is directed edge from a_i to a_{i+1} for every vertex $a_i < max$. It can be seen as injective function $F :: [0...n] \longrightarrow |\mathcal{A}|$

$$(\exists F \in \text{Inj})(\forall x)[x < \text{max} \rightarrow (\exists x'yz)(E(y, z) \wedge F(x, y) \wedge \text{SUC}(x, x') \wedge F(x', z))]$$

The Tool



Input:

- signature σ that contains relational symbols
- $\text{SO}\exists$ formula Φ that encodes **NP problem**
- finite structure \mathcal{A} that encodes **instance**

Output:

- PDDLs for a **fragment** of STRIPS that is **decidable in NP**

Guarantees:

- runs in polytime for **fixed** Φ
- output is no **harder** than input (complexity wise)

Related Work

- DATALOG-like specification of NP problems into SAT (Cadoli & Schaerf, 2005)
 - we are targetting STRIPS
 - we would like to go beyond NP
- Framework for describing problems based on the Model Extension (MX) (Mitchell & Ternovska, 2005)
 - translated problems are solvable using planning technology

Translation

Two Steps

Translation divided in two steps:

- generation of PDDL **domain**
- generation of PDDL **problem instance**

Can be thought as two functions:

$\mathcal{D} : \text{Signatures} \times \text{SO}\exists \rightarrow \text{PDDL Domains}$

$\mathcal{I} : \text{Signatures} \times \text{SO}\exists \times \text{STRUC} \rightarrow \text{PDDL Instances}$

Different Translations

Translations that aim different planners:

- for sequential planners
- for parallel planners
- for optimal sequential planners

Translation Used in Experiments

Domain $\mathcal{D}(\sigma, \Phi)$:

- Φ assumed to have negations **only** at literal level
- two predicates for each relational symbol P
 - $P(?x)$
 - $\text{not-}P(?x)$
- operators that add **positive** fluents for **quantified relations**
- for each FO subformula θ of Φ , except literals, there are
 - fluent that denote the validity of θ wrt extended \mathcal{A}
 - operators that add the fluent

Translation Used in Experiments

Instance $\mathcal{I}(\sigma, \Phi, \mathcal{A})$:

- objects for each element in universe $|\mathcal{A}|$
- initial situation:
 - fluents for interpretations in \mathcal{A}
 - all 'not-' fluents for quantified relations (SO)

Example: SAT – Fluents

$$\underbrace{(\exists T)}_{\text{SAT}} (\forall c) (\exists x) [(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]$$

- $(T \text{ ?x})$ (not- $T \text{ ?x}$)

Example: SAT – Fluents

$$(\exists T)(\forall c)(\exists x)[\underbrace{(P(x, c) \wedge T(x))}_{\text{Fluent}} \vee (N(x, c) \wedge \neg T(x))]$$

- $(T \text{ ?x})$ (not- $T \text{ ?x}$)
- $(P \text{ ?x ?c})$

Example: SAT – Fluents

$$(\exists T)(\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee \underbrace{(N(x, c) \wedge \neg T(x))}]$$

- (T ?x) (not-T ?x)
- (P ?x ?c)
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Example: SAT – Fluents

$$(\exists T)(\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee \underbrace{(N(x, c) \wedge \neg T(x))}]$$

- (T ?x) (not-T ?x)
- (P ?x ?c)
- (N ?x ?c)
- (holds_and_6 ?x ?c)

Example: SAT – Fluents

$$(\exists T)(\forall c) \underbrace{(\exists x)[(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]}$$

- (T ?x) (not-T ?x)
- (P ?x ?c)
- (N ?x ?c)
- (holds_and_6 ?x ?c)
- (holds_exists_8 ?c)

Example: SAT – Actions

$$\underbrace{(\exists T)}(\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]$$

```
(:action set_true_11
  :parameters      (?x0)
  :precondition    (and (guess) (not_T ?x0))
  :effect          (and (T ?x0) (not (not_T ?x0)))
)
```

Example: SAT – Actions

$$(\exists T)(\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee \underbrace{(N(x, c) \wedge \neg T(x))}]$$

```
(:action establish_and_6
:parameters      (?x ?c)
:precondition    (and (proof) (N ?x ?c) (not_T ?x))
:effect          (holds_and_6 ?x ?c)
)
```

Example: SAT – Actions

$$(\exists T)(\forall c) \underbrace{(\exists x)[(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]}$$

```
(:action establish_exists_8
:parameters    (?x ?c)
:precondition  (and (proof) (holds_or_7 ?x ?c))
:effect        (holds_exists_8 ?x)
)
```

Example: SAT – Actions

$$(\exists T) \underbrace{(\forall c)(\exists x)[(P(x, c) \wedge T(x)) \vee (N(x, c) \wedge \neg T(x))]}$$

```
(:action prove_forall_9_1
  :precondition (and (proof) (holds_exists_8 zero))
  :effect (holds_forall_9 zero)
)

(:action prove_forall_9_2
  :parameters (?y1 ?y2)
  :precondition (and (proof) (suc ?y1 ?y2)
                    (holds_forall_9 ?y1) (holds_exists_8 ?y2))
  :effect (holds_forall_9 ?y2)
)
```

Formal Properties

Let $\mathcal{G}(\text{dom}, \text{ins})$ be PDDL **grounding function** (generates STRIPS)

Define $f_{\sigma, \Phi} : \text{STRUC}[\sigma] \rightarrow \text{STRIPS}$ as

$$f_{\sigma, \Phi}(\mathcal{A}) = \mathcal{G}(\underbrace{\mathcal{D}(\sigma, \Phi)}_{\text{domain}}, \underbrace{\mathcal{I}(\sigma, \Phi, \mathcal{A})}_{\text{instance}})$$

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Thm: $f_{\sigma, \Phi}$ is a **polytime reduction** from $\text{MOD}[\Phi]$ into a fragment of STRIPS that is **decidable in NP**

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Thm: $f_{\sigma, \Phi}$ is a **polytime reduction** from $\text{MOD}[\Phi]$ into a fragment of STRIPS that is **decidable in NP**

Thm: if $f_{\sigma, \Phi}(\mathcal{A})$ has plan, it has one with **parallel makespan** at most $\text{MkSp}_{\Phi}(\mathcal{A}) = \mathcal{O}(\|\mathcal{A}\| \cdot \|\Phi\|)$ (i.e. linear in $\|\mathcal{A}\|$ for **fixed** Φ)

Experiments

Performed in Xeon 1.86GHz CPUs with 2GB of RAM

Jussi Rintanen's M planner (SAT-based planner)

Domains (all NP-Complete):

- SAT
- Clique
- Directed Hamiltonian Paths
- 3-Dimensional Matching
- 3-Colorability
- k -Colorability
- Chromatic Number (beyond NP)

Instances:

- SAT: from SATLIB w/ satisfiable and unsatisfiable instances
- others: randomly generated w/ positive and negative instances

Summary of Results

- total of 1,920 **problem instances**
- total of 1,614 **solved instances**
 - 706 on positive side (input structure satisfies formula)
 - 908 on negative side (input structure doesn't satisfy formula)
- M solved 84.06% of the benchmark

Domains

	N^*/N	#pos.	#neg.	avg. time
SAT				
uuf50	40/40	0	40	548.5
uuf75	1/40	0	1	1,746.4
Clique				
25-3	40/40	30	10	111.9
25-4	40/40	18	22	231.0
25-5	39/40	10	29	387.5
25-6	36/40	8	28	394.1
Hamiltonian Path				
30	22/40	20	2	629.1
3-dimensional Matching				
20	13/40	13	0	1,191.0
25	0/40	0	0	—
3-colorability				
50	40/40	1	39	196.7

Chromatic Number

instance	χ	k -colorability						
		1	2	3	4	5	6	7
10-0.75-1	5	2	2	6	101	3		
10-0.75-2	5	1	2	2	6	4		
10-0.85	7	2	2	3	6	4	1,265	4
15-0.25	2	27	62					
15-0.60	5	27	29	54	118	72		
15-0.70	6	28	28	33	47	329	67	
20-0.10	3	214	350	705				
20-0.25	4	211	272	1,261	837			

Discussion

- tool produces PDDLs from **declarative** descriptions
- can be thought as **automatic generation of reductions**
- different translations available, **only one implemented**
- different applications for the tool
- **not every NP problem has a nice formula!**

Future:

- improve tool by incorporating types, other translations, ...
- aim at other complexity classes (fragments of logic)

Thanks!